Appendix A

Fault zone properties and reflectivity

According to the review of Mooney and Meissner (1992), the correlation of high upper crustal reflectivity with mylonite zones was established by tracing this reflectivity to the outcrop of mylonites (e.g., Christensen and Szymanski, 1988). The structural and petrologic characteristics of these mylonite zones indicate that they are representative of mid- to lower crustal mylonite zones. These studies demonstrate that fault zone reflectivity does not result simply from reflections from the top and bottom of the fault, but that reflections with the highest amplitudes can originate from within the fault zone. These reflections are caused by internal fault zone structure and a complex interaction of lithologic layering. These layers may often have varying amounts of seismic anisotropy resulting from ductile strain (McCaffree and Christensen, 1993).

This reflective character of mylonite zones, mostly attributed to the preferential orientation of mineralogical components producing velocity anisotropy, has been widely studied (e.g., Christensen, 1989). As an example of material and elastic properties for a mylonite zone, see Table B of Appendix B for the mylonite of the Brevard fault zone in the southern Appalachians (Christensen and Szymanski, 1988; McCaffree and Christensen, 1993).

Appendix B

Elastic properties and perturbations of Lamé parameters

Extensive tables of elastic properties of rocks and minerals are easily available in the works of Birch (1966), Sumino and Anderson (1984), and Schön (1996, chapter 6). Concerning Lamé parameters, there is abundant data for μ (rigidity), because of its simple physical attribute as the shear modulus. The second Lamé parameter, λ, is most significant in combination with other terms (e.g., in terms of μ and k, the bulk modulus or incompressibility, $\lambda = k - 2\mu/3$) and does not have a simple physical meaning (but it greatly simplifies Hooke's law). Because of this, it is difficult to grasp the meaning of $\Delta\lambda$ images, like those shown in Chapters 3 and 4.

For the sake of illustration, Table A contains material and elastic properties for Figs. 3.4, 3.5, 3.6, and 3.15. For the elastic properties, I utilize the most commonly used unit, Gigapascals (GPa), where 1 Pascal (Pa) = 1 N m^{-2} = 1 kg m^{-1} s^{-2}. Table B contains material and elastic properties for the mylonite of the Brevard fault zone in the southern Appalachians (Christensen and Szymanski, 1988, Table 2; McCaffree and Christensen, 1993, Table 1). Velocities (α and β)

and densities (ρ) are given as mean values for a pressure of 400 MPa, corresponding to intermediate crustal depths. All properties are shown for 13 samples from the Brevard fault zone core at intervals along the core (Christensen and Szymanski, 1988).

For both Tables, the perturbations or relative changes of Lamé parameters between the upper (0) and the lower (1) layer at each interface are given at the row corresponding to the lower layer. Note that my convention here is to use the upper layer (0) as the "background material," and subtract the value of the lower layer from that of the upper one. Of course, this yields negative values for interfaces where moduli increase with depth.

I use the following equations to compute the perturbations or relative changes of Lamé parameters. The values with asterisks correspond to simple, intuitive relative changes, whereas the values without asterisks correspond to the proper expressions obtained by Sato (1984, eq. 8) under the Born approximation:

$$\frac{\Delta\lambda^*}{\lambda_o} = \frac{\lambda_o - \lambda_1}{\lambda_o},$$

$$\frac{\Delta\lambda}{\lambda_o} = \left[2\frac{\Delta\alpha}{\alpha_o} + \frac{\Delta\rho}{\rho_o}\right] + 4\frac{\mu_o}{\lambda_o}\left[\frac{\Delta\alpha}{\alpha_o} - \frac{\Delta\beta}{\beta_o}\right],$$

$$\frac{\Delta\mu^*}{\mu_o} = \frac{\mu_o - \mu_1}{\mu_o},$$

and

$$\frac{\Delta\mu}{\mu_o} = 2\frac{\Delta\beta}{\beta_o} + \frac{\Delta\rho}{\rho_o}.$$

Finally, note that there is a very large relative change of λ (-2200 and -1980) between Layers 2 and 3 in Table A for the contrived model for Fig. 3.6. This is because the V_P/V_S ratio for Layer 2, $V_P/V_S = 1.43$, is lower than typical values. Typical ranges for the V_P/V_S ratio lie between 1.65 and 3.0 for sedimentary rocks (e.g., Schön, 1996, p. 208), and 1.6 and 1.8 for mylonites (McCaffree and Christensen, 1993). Although this may look unrealistic, it helps to emphasize and illustrate the imaging of the $\Delta\lambda$ interface shown in Fig. 3.8 (16 km interface).

Table A

Data for Figures 3.4 and 3.5

Layer Number	α km/s	β km/s	ρ kg/m^3	λ GPa	μ GPa	$\frac{\Delta\lambda^*}{\lambda}$ %	$\frac{\Delta\lambda}{\lambda}$ %	$\frac{\Delta\mu^*}{\mu}$ %	$\frac{\Delta\mu}{\mu}$ %
1	5.0	2.9	3000	24.5	25.2	–	–	–	–
2	6.0	2.9	3000	57.5	25.2	-130.8	-122.2	0	0
1	5.0	2.9	3000	24.5	25.2	–	–	–	–
2	6.0	3.7	3000	24.5	41.7	0	0	-65.4	-57.2

Data for Figure 3.6

Layer Number	α km/s	β km/s	ρ kg/m^3	λ GPa	μ GPa	$\frac{\Delta\lambda^*}{\lambda}$ %	$\frac{\Delta\lambda}{\lambda}$ %	$\frac{\Delta\mu^*}{\mu}$ %	$\frac{\Delta\mu}{\mu}$ %
1	5.0	2.9	3000	24.5	25.2	–	–	–	–
2	5.0	3.5	3000	1.5	36.7	93.9	85.1	-45.6	-41.4
3	6.0	3.5	3000	34.5	36.7	-2200	-1980	0	0
4	6.0	3.5	3600	41.4	44.1	-20.0	-20.0	-20.0	-20.0
5	7.0	4.1	4200	64.6	70.6	-56.0	-51.3	-60.0	-51.0

Table A (Cont.)

Data for Figure 3.15

Layer Name	α km/s	β km/s	ρ kg/m^3	λ GPa	μ GPa	$\frac{\Delta\lambda^*}{\lambda}$ %	$\frac{\Delta\lambda}{\lambda}$ %	$\frac{\Delta\mu^*}{\mu}$ %	$\frac{\Delta\mu}{\mu}$ %
Upper	5.20	3.0	2500	22.6	22.5	–	–	–	–
Fault Zone	4.85	2.8	2500	19.6	19.6	13.2	13.7	12.9	13.3
Lower	5.55	3.2	2500	25.8	25.6	-31.6	-29.5	-30.6	-28.6

Table B

Elastic properties for the data of Christensen and Szymanski (1988)

and McCaffree and Christensen (1993)

for a pressure of 400 MPa

Sample Depth,m	α km/s	β km/s	ρ kg/m^3	λ GPa	μ GPa	$\frac{\Delta\lambda^*}{\lambda}$ %	$\frac{\Delta\lambda}{\lambda}$ %	$\frac{\Delta\mu^*}{\mu}$ %	$\frac{\Delta\mu}{\mu}$ %
29.0	6.303	3.268	2782	53.3	28.6	–	–	–	–
34.7	6.293	3.489	2778	42.4	33.8	20.4	19.6	-18.1	-17.4
43.6	6.273	3.604	2646	35.4	34.4	16.5	19.4	-1.6	-1.8
84.4	6.037	3.290	2803	41.5	30.3	-17.2	-12.9	11.7	11.5
122.0	6.277	3.518	2744	40.2	33.9	3.09	4.1	-11.9	-11.8
126.8	6.347	3.433	2749	45.9	32.4	-14.3	-12.3	4.6	4.6
144.8	6.537	3.471	2784	51.9	33.5	-12.9	-12.1	-3.5	-3.5
167.9	6.383	3.345	2800	51.4	31.3	0.89	1.0	6.6	6.7
170.1	6.100	3.543	2667	32.3	33.5	37.2	56.6	-6.8	-7.1
212.5	6.437	3.588	2727	42.8	35.1	-32.5	-27.3	-4.9	-4.8
244.8	6.343	3.654	2659	36.0	35.5	15.9	18.4	-1.1	-1.2
274.9	6.373	3.599	2700	39.7	35.0	-10.4	-9.4	1.5	1.5
303.9	6.337	3.426	2686	44.8	31.5	-12.8	-10.3	9.8	10.1

Appendix C

Reflections and reflection coefficients

Intuitively, one should expect that, for a simple wave propagation exercise, a continuous distribution of point scatterers must give the same result as, for instance, reflection coefficients from a flat interface. Matson (1996) discussed the equivalence between the "inverse acoustic backscattering view" (e.g., Weglein, 1985; Carrion, 1987) and the "traditional view" (from the Zoeppritz equations; e.g., Young and Braile, 1976; Aki and Richards, 1980, Vol. I, p. 153; Shuey, 1985) of reflections and reflection coefficients. Matson shows, for a two-layer model with a planar interface, that the asymptotic limit of the "inverse acoustic backscattering view" is in fact achieved (see Matson, 1996, Fig. 2). The Born approximate reflection coefficient does approach the proper Zoeppritz value for small values of the $P-$wave velocity ratio (i.e., when $\alpha_0/\alpha_1 \simeq 1$), which is the weak scattering convergence criterion for the forward Born series (Matson, 1996).

One can obtain ballpark estimates of the reflection and transmission coefficients for the "inverse acoustic backscattering view" and put them into the context of the more familiar, "traditional view" of Zoeppritz reflection coefficients. Before doing this, however, it is necessary to give some background about the example I provide below, based on the simple model I used to compute Figs. 3.4 and 3.5.

Figs. 3.4 and 3.5 have the material properties shown in Appendix B for the case of λ and μ perturbations. They both show vertical-component elastic synthetics for a two-layer model. I use a fourth-order finite-difference solution of the elastic wave equation to compute $P-SV$ seismograms (Levander, 1988). The seismic excitation is a vertical-motion line source of an assumed Ricker wavelet with 2 Hz central frequency. For backscattered waves, the source is located at 5 km depth, whereas for forward scattered waves the source is located at 15 km. All four seismograms are recorded on the surface, at a (horizontal) source-receiver distance of 20.8 km. The interface is located at 10 km depth.

The backscattering case shows the first arrival and the early "coda of reflection" (depicting a $P-P$ reflection) for variations in Lamé parameters. The forward scattering case shows the first arrival (depicting a $P-P$ transmission) and the early "coda of transmission" after that. Note the absence of a forward-scattered $S-$wave (i.e., there is no $P-S$ energy) for the $\Delta\lambda$ scattering component despite the nonnormal incidence of energy.

For this model, a backscattered ray (from the 5 km deep source) has an incidence angle at the interface of 54 degrees, and a scattering angle (the angle of aperture between the incident ray and the scattered ray) of 108 degrees. A forward scattered ray has an incidence angle at the interface of 126 degrees, and a scattering angle of 180 degrees (arriving slightly after the first arrival). With these values, one can now obtain ballpark estimates of the reflection and transmission coefficients. In traditional wave propagation analysis, the ratio of the $P-$wave amplitudes of the reflected and incident waves gives the reflection coefficient R_{PP}, and the ratio of the $P-$wave amplitudes of the transmitted and incident waves gives the transmission coefficient T_{PP}.

For the λ perturbation case of Figs. 3.4 and 3.5, the inverse acoustic backscattering view leads to the reflection and transmission coefficients $R_{PP} = 0.44$ and $T_{PP} = 0.44$ (in this case, the coefficients are independent of the scattering angle). I computed both values with the expression provided by Le Bras [1985, eq. 4.17 (see also Wu and Aki, 1985, eq. 9)]:

$$R_{PP} = a_1 \cos\theta - a_2 + 2\frac{\beta_0^2}{\alpha_0^2} a_3 \sin^2\theta,$$

where θ is the scattering angle, and

$$a_1 = 1 - \frac{\rho_1}{\rho_o}, \quad a_2 = 1 - \frac{\lambda_1 + 2\mu_1}{\lambda_o + 2\mu_o},$$

and

$$a_3 = 1 - \frac{\mu_1}{\mu_o}.$$

The Zoeppritz reflection and transmission coefficients for this case, computed with the program of Young and Braile (1976), are $R_{PP} = 0.493$ and $T_{PP} = 1.244$. One can also roughly obtain, directly from the finite-difference seismograms (not shown), that $R_{PP} = 0.67$ and $T_{PP} = 0.88$.

Now, for the μ perturbation case, the inverse acoustic backscattering view leads to the reflection and transmission coefficients $R_{PP} = 0.044$ and $T_{PP} = 0.44$. The Zoeppritz reflection and transmission coefficients for this case are $R_{PP} = 0.229$ and $T_{PP} = 1.326$. Finally, directly from the finite-difference seismograms (not shown), $R_{PP} = 0.21$ and $T_{PP} = 1.25$.

Thus, we can establish, mostly for the λ perturbation case, a reasonably accurate mapping between a scattering model of seismic reflection data and a well known solution for reflection coefficients. The latter represents the way seismologists traditionally think of events as being generated from wave propagation and reflections from interfaces. The former is the scattering theory point of view which forms the basis of my seismic imaging work.