Application Examples

4) Ptroff, UNIX text formatter from Adobe:

```
%!PS-Adobe-1.0
%%Creator: shake:louie (John Louie)
%%Title: stdin
%%CreationDate: Tue Jan 26 18:31:13 1993
%%DocumentFonts: Times-Roman Times-Italic Times-Bold Symbol Times-Roman DIThac
%%Pages: (atend)
%%EndComments
% Start of pscat.pro -- prolog for troff translator
. . .
save /pscatsave exch def
/$pscat 50 dict def
$pscat begin
/fm [1 0 0 1 0 0] def
/xo 0 def /yo 0 def
/M /moveto load def
/R /show load def
/S {exch currentpoint exch pop moveto show}def
. . .
%%EndProlog
%%Page: ? 1
BP
1 F
72 Z
598 462(We)U
732(will)S
878(employ)S
1128(the)S
1249(formalism)S
. . .
1644 924(\256)U
1896(\256)S
2 F
84 Z
1691 978(t)U
4 F
1756(=)S
3 F
1831(L)S
4 F
(D)R
. . .
1724(Clayton,)S
1994(1984\).)S
EP
%%Trailer
pscatsave end restore
%%Pages: 1
```

We will employ the formalism elicited by Hearn and Clayton (1986a,b) and by Humphreys and Clayton (1988). We divide our two-dimensional map of refractor velocity into discrete blocks, each having a uniform slowness (reciprocal of the velocity) perturbation. Straight rays between epicenters and stations cross these blocks. To compute a forward model of synthetic ray delay times from a slowness map,we use:

$$\vec{\Delta t} = \mathbf{L}\vec{\Delta s} \quad ;$$

where $\vec{\Delta t}$ is a vector holding the delay times Δt_r for each ray r, $\mathbf{L}$ is a matrix of ray lengths l_{rb} of each ray in each block b, and $\vec{\Delta s}$ is a vector of slowness perturbations Δs_b for each block. Assuming instead that we have delay time data, we can back project the delays to estimate an approximation of the slowness map:

$$\vec{\Delta s} = \mathbf{D}^{-1}\mathbf{L}^{\mathrm{T}}\vec{\Delta t} \quad ,$$

where $\mathbf{D}$ is a matrix containing only the diagonal of $\mathbf{L}^{\mathrm{T}}\mathbf{L}$. Such back projections typically only account for a minor portion of the variance of the ray delay data (Humphreys and Clayton, 1988). In addition, the imperfect distribution and anisotropy of our ray set will tend to spread anomalies out along predominant ray directions.

We can attempt to improve our inversion through Jacobi iteration, where further approximations $\vec{\Delta s}^{(k)}$ result from the correction of a previous model $\vec{\Delta s}^{(k-1)}$:

$$\vec{\Delta s}^{(k)} = \mathbf{D}^{-1}\mathbf{L}^{\mathrm{T}}\vec{\Delta t} + (\mathbf{D} - \mathbf{L}^{\mathrm{T}}\mathbf{L})\vec{\Delta s}^{(k-1)} \quad .$$

Each correction is essentially the inverse of the difference between the ray delay data and the delays forward-modeled through the previous model. Comer and Clayton (1984) and Hager et al. (1985) showed that this iteration will converge to the least-squares solution solution for $\vec{\Delta s}$ (Aki and Lee, 1976):

$$\vec{\Delta s} = (\mathbf{L}^{\mathrm{T}}\mathbf{L})^{-1}\mathbf{L}^{\mathrm{T}}\vec{\Delta t} \quad .$$

We must use tomography because $\mathbf{L}^{\mathrm{T}}\mathbf{L}$ is too large to invert directly. For our case, to cover the Sierra and the surrounding events and stations at 20 km resolution would require at least 700 blocks, giving $\mathbf{L}^{\mathrm{T}}\mathbf{L}$ almost half a million elements.

Hager et al. (1985), Hearn and Clayton (1986a,b), and Humphreys and Clayton (1988) emphasize the importance of ray coverage in controling the quality of inverted as well as iterated and back-projected models. Direct inversion or its approximation through iteration will drastically increase the inverted proportion of the data variance and focus the anomaly spreads over the simple back projection. However, these effects remain prominent even in a perfect least-squares inversion if the ray set is significantly heterogeneous or anisotropic (Hearn and Clayton, 1986a,b; Fawcett and Clayton, 1984).